

CHAPTER 3—THE INFINITE SLOPE MODEL

3.1 Description

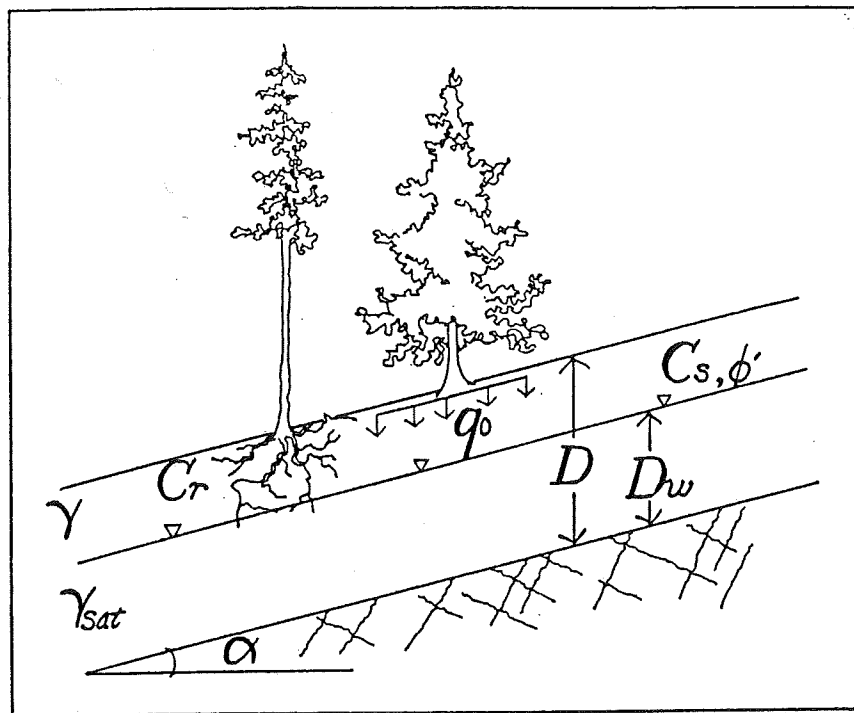
LISA uses the infinite slope stability model to calculate the factor of safety. The infinite slope model geometry and equation are shown in figure 3.1. Appendix A gives the derivation of the infinite slope equation. We selected the infinite slope model primarily because the model's simplicity allows for easy use in Monte Carlo simulation, not because of its accuracy. However, experience has shown that if used carefully, it does adequately analyze for planning purposes the most common failure types found in the mountainous West—debris flows and debris avalanches characterized by the failure of a soil mantle that overlies a sloping drainage barrier (Gray and Megahan 1981; Prellwitz and others 1983; Sidle and others 1985; Wu and others 1979). The drainage barrier may be bedrock or a denser soil mass. The factor of safety calculated by the infinite slope equation corresponds closely with that calculated for translational failures using a more rigorous method of slices, such as Janbu's Simplified Method. In general, the infinite slope equation, and therefore LISA, does not adequately analyze deep-seated rotational failure modes. However, the probability of rotational slope failures may be reasonably estimated using LISA if conditions that exist at the center of gravity of a failure mass are used in the analysis. The procedure for estimating the conditions at the center of gravity is described in detail by Prellwitz (1988), and an example application is given by Ristau (1988).

3.2 Assumptions

The infinite slope model relies on several simplifying assumptions. First, the failure plane and the groundwater (phreatic) surface are assumed to be parallel to the ground surface. The drainage barrier and ground surface often are found to be nearly parallel on colluvial slopes. Also, a large hydraulic conductivity contrast between the soil and drainage barrier can cause groundwater flow to be nearly parallel to the drainage barrier. Therefore, the conditions of parallelism often are approximately met. However, the user should be aware that parallel seepage may not be the case, and if not, the factor of safety may be significantly overestimated or underestimated, depending on the actual seepage direction (Iverson and Major 1987, 1986).

Second, the failure plane is assumed to be of infinite extent. Of course, in nature the failure plane does extend to the ground surface. Therefore, values for root strength and soil shear strength that reflect conditions along the true failure plane, not just along the drainage barrier, should be used. For example, when the infinite slope failure plane is beneath the root zone, implying no root strength, some root strength still should be used in the analysis to account for the true failure plane passing through the root zone to the ground surface along the lateral and head scarps. The values of root strength used should, however, be less than if the failure plane passed entirely through the root zone. Suggested root strength values for these different conditions are given in section 5.3.4.

Third, only a single soil layer is considered. In the case of multiple layers, the soil shear strength values occurring at the base should be given the most weight, but as with root strength, values should be adjusted (weighted) to account for the shear strength along the entire failure plane as it extends to the ground surface. For example, suppose 80 percent of the failure plane passed through soil



$$FS = \frac{C_r + C'_s + \cos^2 \alpha [q_0 + \gamma(D - D_w) + (\gamma_{sat} - \gamma_w)D_w] \tan \phi'}{\sin \alpha \cos \alpha [q_0 + \gamma(D - D_w) + \gamma_{sat} D_w]}$$

where FS = factor of safety

α = slope of the ground surface, degrees

D = total soil thickness, ft

D_w = saturated soil thickness, ft

C_r = tree root strength expressed as cohesion, psf

q_0 = tree surcharge, psf

C'_s = soil cohesion, psf

ϕ' = effective internal angle of friction, degrees

γ_d = dry soil unit weight, pcf

γ = moist soil unit weight, pcf

γ_{sat} = saturated soil unit weight, pcf

γ_w = water unit weight, pcf

Figure 3.1—The infinite slope equation and variables used in LISA.

with $C'_s = 20$ psf and $\phi' = 36^\circ$, and 20 percent through soil with $C'_s = 120$ psf and $\phi' = 22^\circ$. The weighted values then would be:

$$C'_s = 0.8(20 \text{ psf}) + 0.2(120 \text{ psf}) = 40 \text{ psf}$$

$$\phi' = 0.8(36^\circ) + 0.2(20^\circ) = 33.2^\circ$$

And last, the infinite slope equation is a two-dimensional analysis. Thus, the user must assume that a two-dimensional analysis is appropriate. Comparison of the infinite slope with a three-dimensional block model (Burroughs 1984) shows that the infinite slope model gives the same answer for blocks with widths greater than about 25 to 30 feet. Therefore, a two-dimensional analysis is most appropriate for wide blocks where resistance along failure sides is not significant relative to resistance along the base. If failures are narrower, the infinite slope model is conservative (it calculates lower factors of safety than does a three-dimensional analysis). A Monte Carlo simulation program using the three-dimensional model (called 3DLISA) is currently under development and evaluation at the Intermountain Research Station in cooperation with the University of Idaho and the Bureau of Land Management.

3.3 Sensitivity to Input Values

A sensitivity analysis of the infinite slope model is helpful to identify the most important variables and thus guide the user in expending time and money collecting information. One method for evaluating the sensitivity of the factor of safety (FS) to each variable has been outlined by Simons and others (1978):

1. Select a realistic range of values for each input variable.
2. Calculate a base FS value using some central value for each variable, such as the mean, median, or mode value.
3. Vary the value for one input variable at a time over the range of realistic values and compute the FS values.
4. Plot the percentage of change in FS ($\% \Delta FS$) relative to the base value against the percentage of change in each input variable relative to the central value ($\% \Delta X$), where the percentage of change is calculated as:

$$\% \Delta FS = \frac{FS \text{ using } x_i - FS \text{ using central } x}{FS \text{ using central } x} \times 100\%$$

$$\% \Delta X = \frac{x_i - \text{central } x}{\text{central } x} \times 100\%$$

Figure 3.2 is a sensitivity plot for a selected set of central values. It is obvious from this figure that increasing soil and root strength will increase the FS , and increasing slope and groundwater-soil depth ratio (or groundwater height) will decrease the FS . Generally, the FS is most sensitive to slope and insensitive to soil unit weight, soil moisture content, and tree surcharge. (FS is so insensitive to the last three factors that they are not even shown on fig. 3.2.) Therefore, it is important to have good field estimates of slope, while unit weight, moisture content, and tree surcharge values can be estimated from the literature.

The relative sensitivity of the FS to the other variables will change depending on the central values selected. This is illustrated by figure 3.3, in which only the central value for soil depth has been changed from 10 feet in figure 3.2 to 2 feet in figure 3.3. The FS becomes more sensitive to soil and root cohesions and less sensitive to groundwater-soil depth ratio and ϕ' when the central value for soil depth is decreased. The sensitivity of FS to soil depth is discussed in greater detail below.

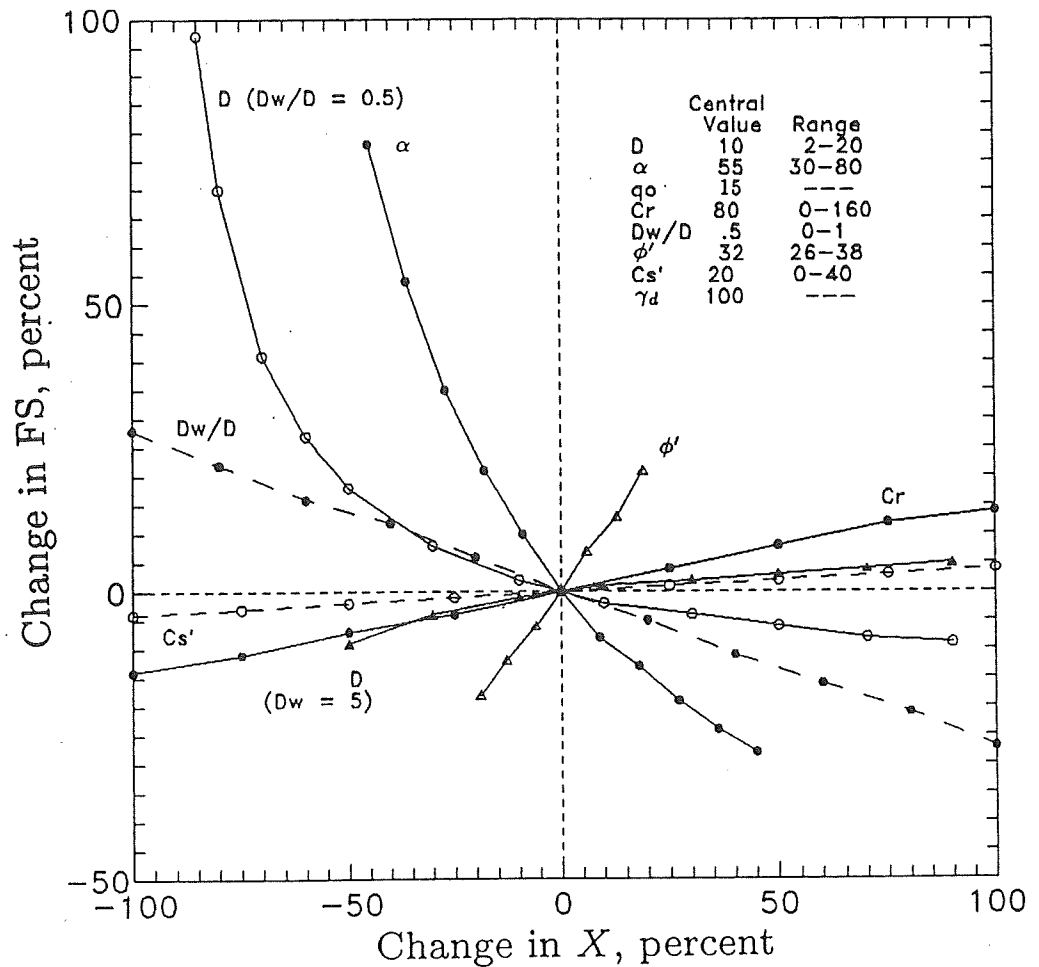


Figure 3.2—Example sensitivity plot for the infinite slope equation with central soil depth equal to 10 feet.

Other important sensitivity trends and interdependencies between variables should be noted.⁴ Figure 3.4 shows that soil and root cohesions ($C_r + C_s$) affect the factor of safety more on thin soils than on thick soils.⁵ Another study (Sidle 1984a) shows that the sensitivity of FS to $C_r + C_s$ is even more pronounced on steep slopes, particularly when the soils are saturated. Thus, altering C_r through timber harvest would affect the stability of thin, steep sites more than thick, gentle sites. Conversely, ϕ' affects the FS more on thick soils (particularly with gentle slopes) than on thin soils (fig. 3.5). These trends should be expected, because frictional strength is more important in conditions of high

⁴Unless otherwise stated, the central values for figures 3.4 to 3.7 are the same as those used in figure 3.2. These figures show the percentage of change in FS relative to the lowest value of X used, rather than to the central value. Plotting in this fashion makes the trends easier to see.

⁵The resisting force in the infinite slope equation is expressed as: $S = C_r + C_s' + \sigma_n' \tan \phi'$. Because soil and root cohesions are added, the sensitivity of FS to each is the same. Thus, the sensitivity to cohesion, irrespective of whether it is from the soil or roots, can be examined by looking at the sum.

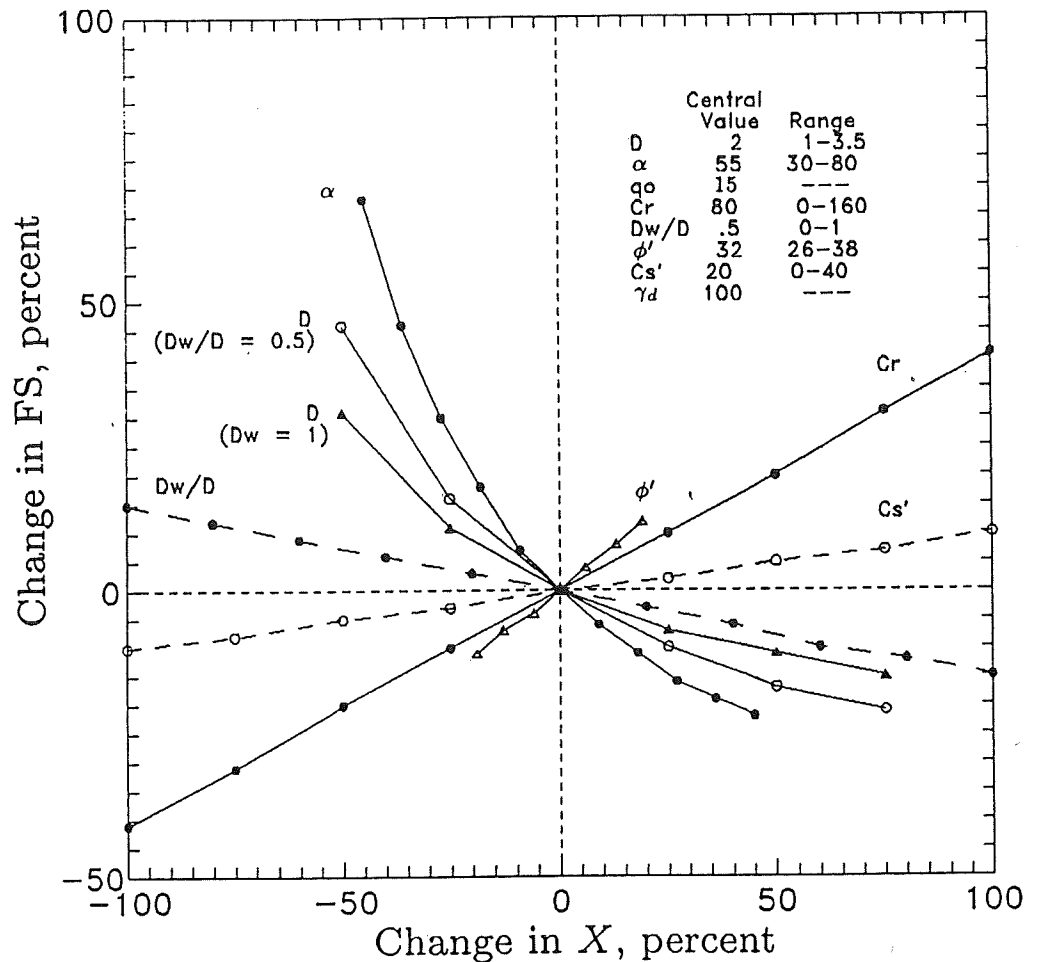


Figure 3.3—Example sensitivity plot for the infinite slope equation with central soil depth equal to 2 feet.

normal stress, and cohesive strength is more important in conditions of low normal stress.

The effect of soil depth (D) on the FS depends on (1) whether or not there is soil or root cohesion and (2) how groundwater is handled in the analysis; that is, the effect of D on FS is different when D_w/D is held constant as D is varied than when D_w is held constant, because when D_w/D is held constant, D_w also varies. Although LISA uses D_w/D , it is informative to note the effects on FS caused by changing D with D_w held constant. The relative magnitude of these effects depends on slope, but the same trends occur on slopes between 20 and 150 percent, the range investigated by the authors.

Figure 3.6 shows the effects of changing D when there is *no* cohesion ($C_r + C'_s = 0$). Three observations can be made:

- When there is *no* groundwater ($D_w/D = 0$), there is *no change* in the FS as D varies. The change in driving force directly balances the change in resisting force. (The infinite slope equation for this case simplifies to $FS = \tan \phi' / \tan \alpha$, showing directly that FS is independent of D .)

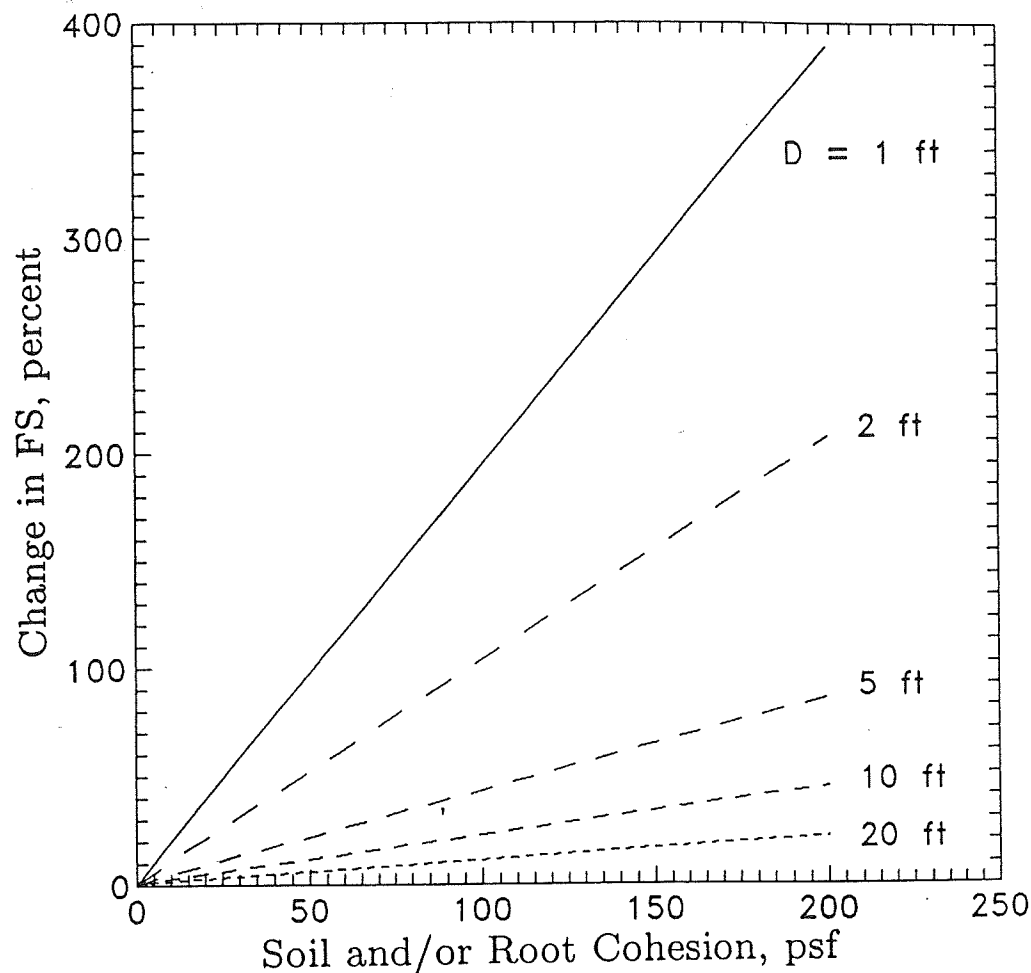


Figure 3.4—Sensitivity of FS to soil and root cohesion at various soil depths.

- When D_w/D is held constant at any value greater than zero, FS decreases slightly with increasing D .
- When D_w is held constant at any value greater than zero, FS increases with increasing D .

Figure 3.7 shows the effects of changing D when there *is* cohesion ($C'_s + C_r > 0$). The variation in FS with changing D is quite different than when $C'_s + C_r = 0$.

- When there is *no* groundwater ($D_w/D = 0$ or $D_w = 0$), there is a fairly large decrease in FS with increasing D .
- When D_w/D is held constant at any value greater than zero, there is even greater decrease in FS with increasing D .
- When D_w is held constant, different effects on the FS with changing D are observed. For every set of central values, there will be one value for D_w for which there will be *no* change in FS as D varies (3.4 ft in fig. 3.7). For D_w values greater than this equilibrium value of D_w , the FS will *in-*

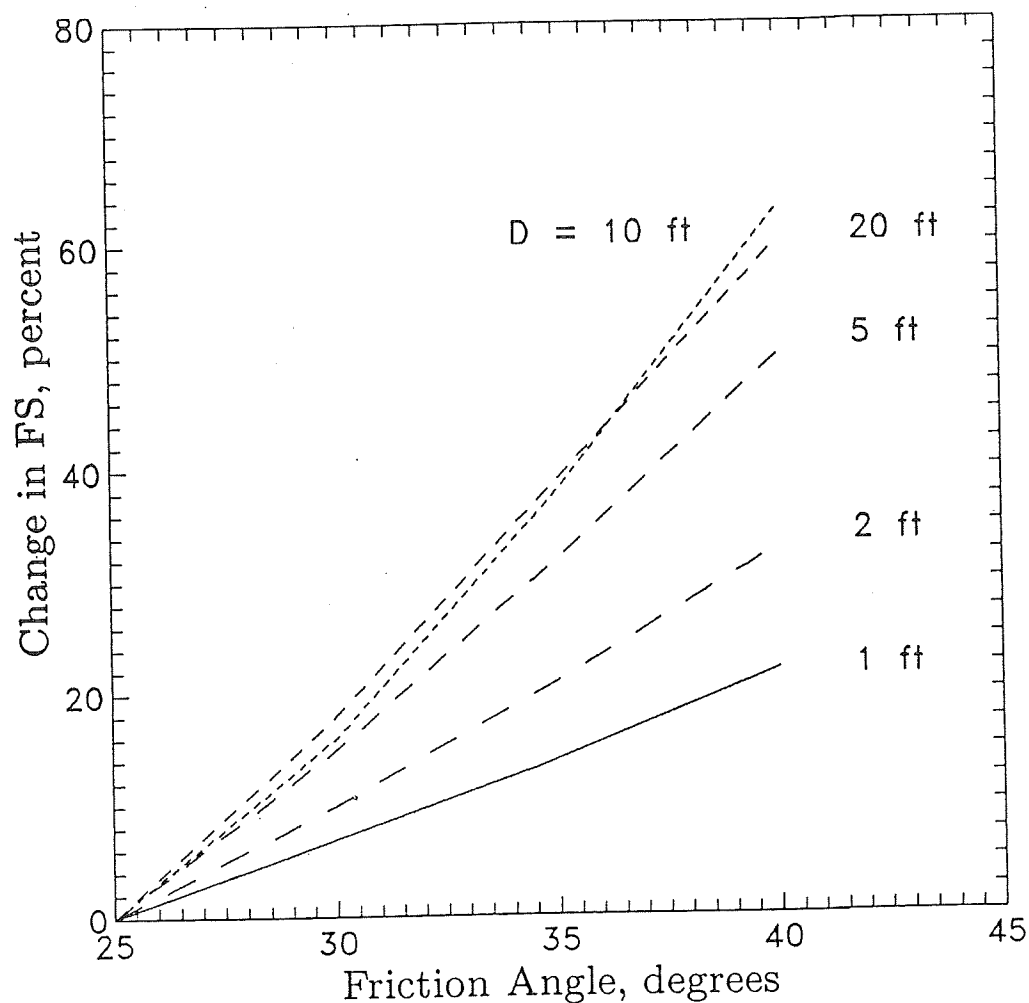


Figure 3.5—Sensitivity of FS to friction angle at various soil depths.

crease as D increases. For D_w values less than this equilibrium value, the FS will decrease as D increases.

Thus, the user should appreciate that whether the FS increases or decreases with changing soil depth, as well as the sensitivity of the FS to soil depth, depends on the groundwater and cohesion ($C'_s + C_r$) values used. However, in general, it is wise to consider the FS sensitive to D and plan on spending some effort in obtaining reliable field estimates for D values.

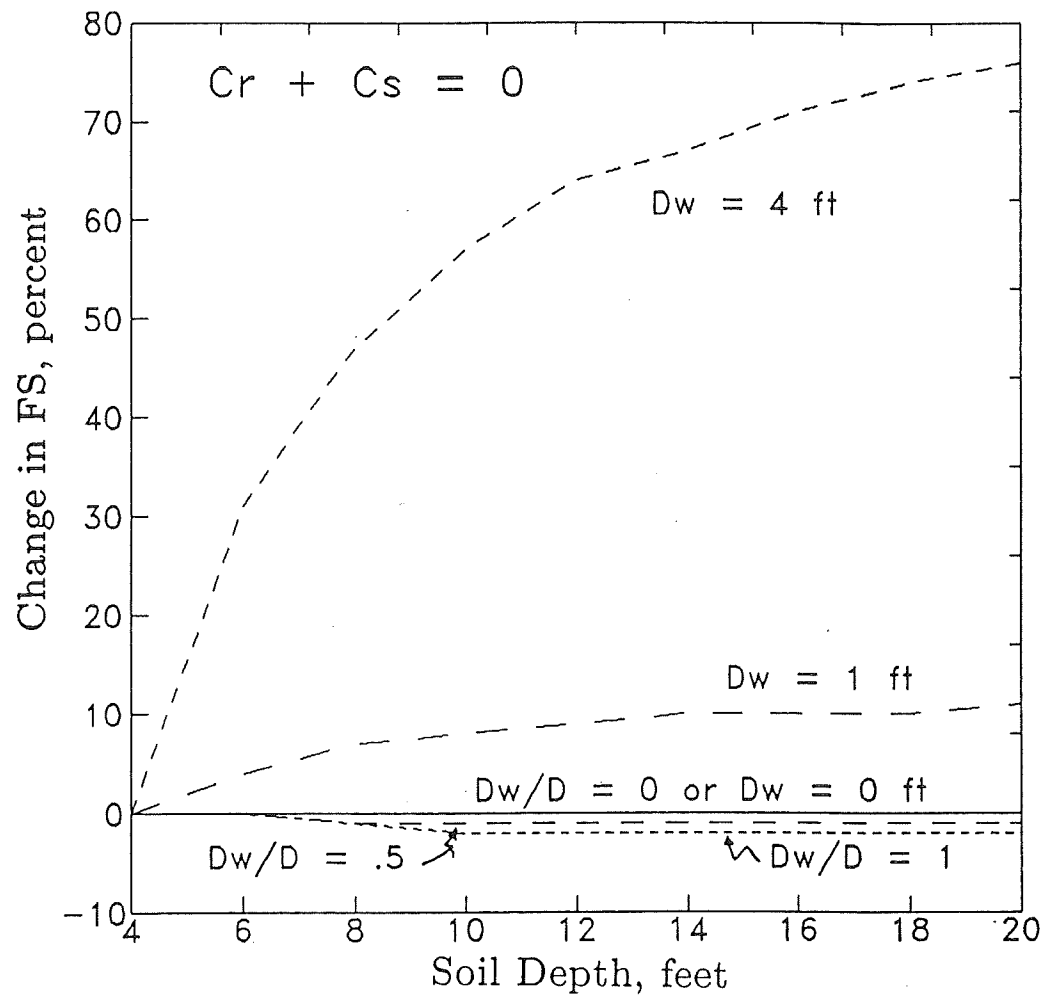


Figure 3.6—Sensitivity of FS to soil depth when there is no soil or root cohesion. Sensitivity varies depending on how groundwater is handled.

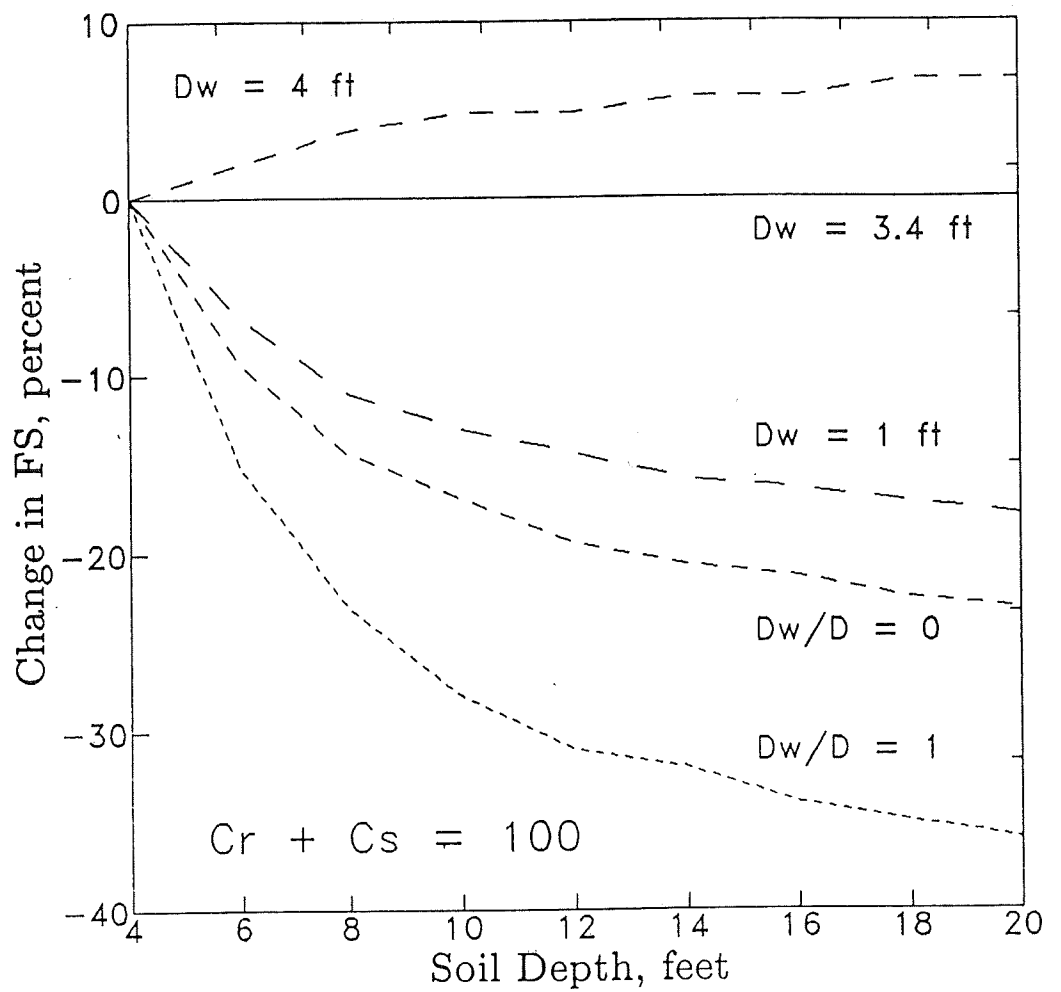


Figure 3.7—Sensitivity of FS to soil depth when there is soil or root cohesion or both. Again, sensitivity varies depending on how groundwater is handled.

CHAPTER 4—HOW THE LISA PROGRAM WORKS (THE INSIDE NUTS AND BOLTS)

4.1 Overview

In general, the operation of LISA is as follows:

1. The user selects a distribution type for each input parameter in the infinite slope equation and then enters the values to describe that distribution. The user may choose a constant value or a uniform, normal, lognormal, triangular, beta, or relative-frequency histogram distribution. A bivariate-normal distribution also may be selected for C'_s and ϕ' .
2. LISA generates a column of up to 1,000 values for each parameter. The number of values is specified by the user. The various procedures for simulating values from the distributions are beyond the scope of this paper, but procedures can be found in Abramowitz and Stegun (1965), Hall and Kendall (1992), Iman and Shortencarier (1984), Newendorp (1975), and Rubinstein (1981). A frequency histogram of the 1,000 values for each parameter will closely match the shape of the distribution specified by the user, but the 1,000 values are generated in a random order (unless they are correlated to another input parameter as discussed in section 4.2). LISA displays the minimum, maximum, mean, and standard deviation for each variable as the values are generated.
3. LISA then calculates the factor of safety for each set of generated values. The result is 1,000 possible realizations of the factor of safety, with relative frequencies being a result of the distributions used for each input variable. The minimum, maximum, mean, and standard deviation for the factor of safety and probability of failure are displayed.
4. The user then may view the frequency histogram of the factor of safety values and of the values simulated for each variable, and may view scatter plots of any pair of variables, or of a variable and the factors of safety.

Detailed descriptions of LISA operations are found in Part 2—Program Operation.

4.2 Correlation Between Variables

Some of the stochastic variables in the infinite slope equation are not independent. The relationship between these variables must be accounted for to achieve a realistic simulation of FS values. The variables treated as dependent by LISA are C'_s and ϕ' , and γ_d and ϕ' .

Although there exists some contradiction in the literature, C'_s and ϕ' generally are considered to be inversely related, as illustrated in figure 4.1. Correlation coefficient (r) values of -0.2 to -0.85 have been reported (Cherubini and others 1983). Figure 4.2 illustrates how treating C'_s and ϕ' as independent variables could result in simulating unrealistic values of soil shear strength. Illustrated are three sets of shear strength tests on a particular soil, resulting in three Mohr-Coulomb failure envelopes that clearly show an inverse relationship between C'_s and ϕ' . If LISA selected values of C'_s and ϕ' independently, the highest value for each could be selected from the test data (C'_{s3} with ϕ'_1), and the upper dashed failure envelope shown in figure 4.2 could result. Obviously this failure envelope is outside the possibilities given by the test data and would result in shear strength values that are too high. Similarly, shear strength values that are too low also could be simulated using C'_{s1} with ϕ'_3 as illustrated by the lower dashed envelope in figure 4.2.

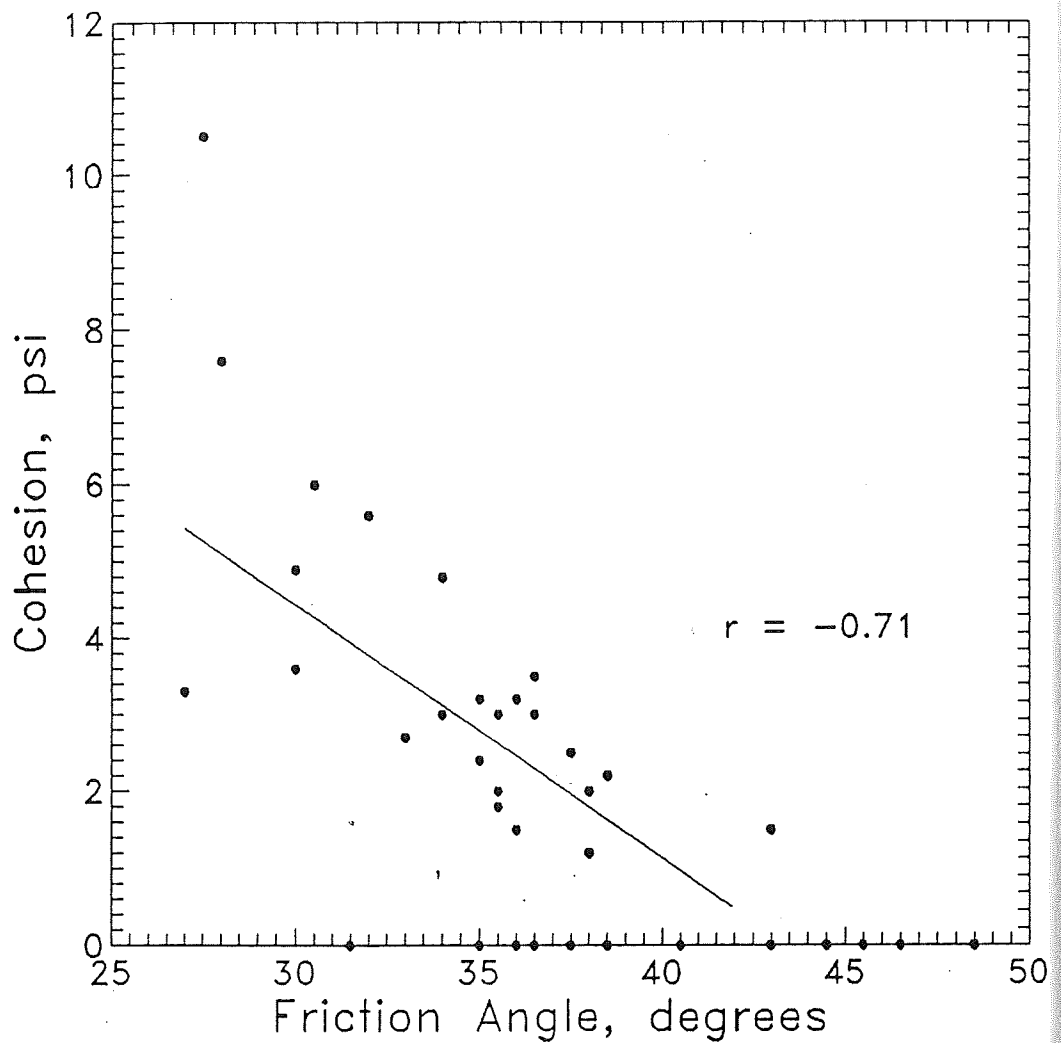


Figure 4.1—Illustration of the inverse relationship between C'_s and ϕ' (data from Hampton and others 1974).

Figure 4.3a and 4.3b contrasts how larger negative values of r act to reduce the variance of simulated shear strength. Values for r may be obtained from laboratory data or estimated from the literature. Section 5.3.5.6 describes how to obtain values for r .

The second relationship considered by LISA is the positive correlation that exists between γ_d and ϕ' . Figure 4.4 shows this correlation for a decomposed granitic soil. The correlation coefficient for this data set is $+0.79$. LISA handles this relationship simplistically by using the same random number to sample from the univariate distributions for γ_d and ϕ' . Therefore, when a high value is sampled for γ_d , a high value is sampled for ϕ' to model the desired proportional relationship. This method produces correlation coefficient between γ_d and ϕ' of 0.95 to 1.0 (with 1.0 occurring when the same distribution type is used for both variables). This degree of correlation is much greater than is found in nature. However, because the infinite slope equation is insensitive to γ_d , the probability of failure values are affected only slightly (usually reduced slightly).

The same random number is *not* used to sample values for γ_d and ϕ' when using the bivariate normal PDF for C'_s and ϕ' . The reason for this is that the bivariate normal would most likely be used to model the shear strength of over-

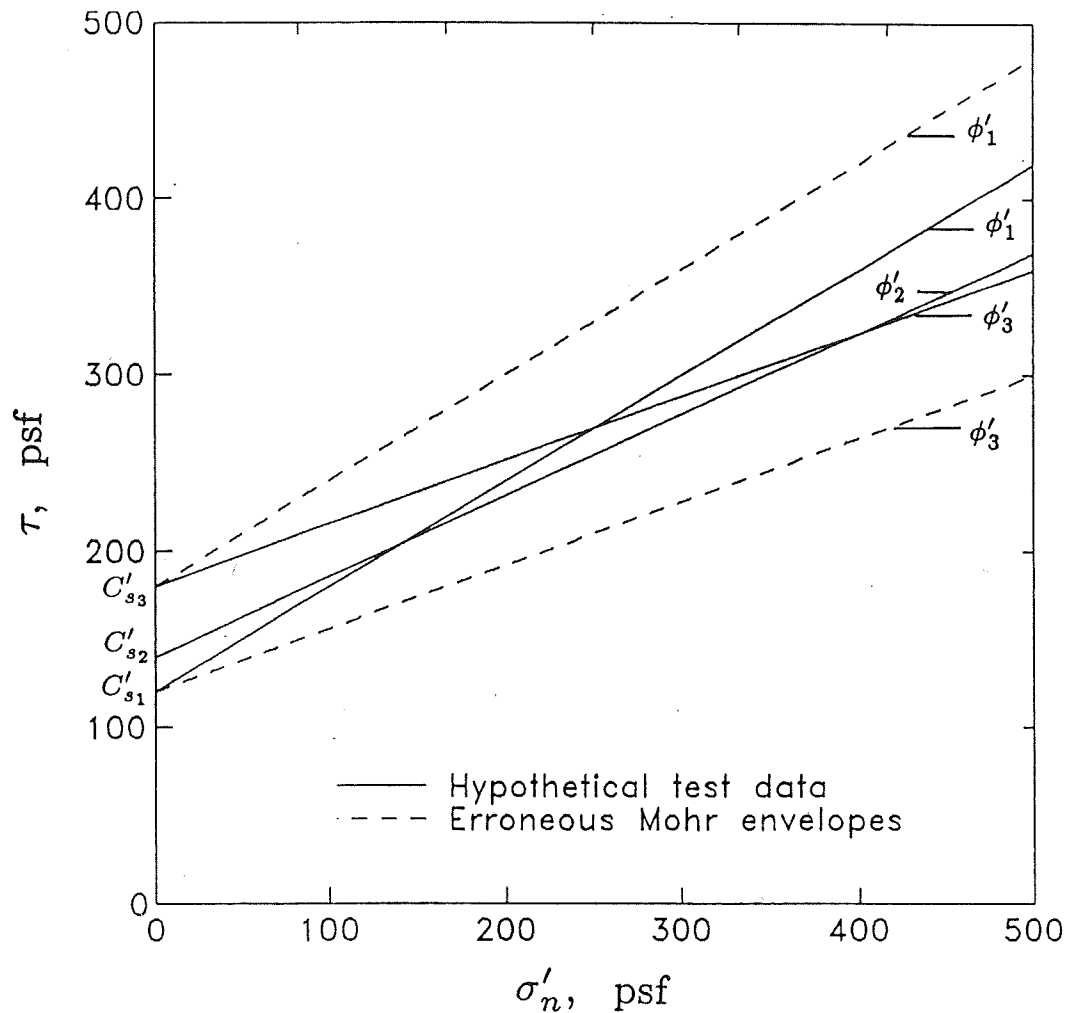


Figure 4.2—Independent sampling of C'_s and ϕ' could result in unrealistic values of shear strength as illustrated by the dashed lines.

consolidated clay which typically shows a C'_s - ϕ' correlation due to curvature of the Mohr-Coulomb failure envelope (see section 5.3.5.3.2). Because of this curvature, it is unclear whether overconsolidated clay will exhibit a correlation between γ_d and ϕ' .

Your field experience may lead you to believe that other variables in the infinite slope equation are correlated. For example, an inverse relationship between soil depth and ground slope is commonly observed. However, it is difficult to obtain a functional relationship that can be used to simulate this correlation without significant amounts of data. A correlation between variables can be accounted for somewhat by more detailed mapping of sites and use of distributions for each site which reflect the observed correlation. Figure 4.5 illustrates distributions for two hypothetical sites in a particular study area which reflect an inverse relationship between soil depth and slope. For individual Monte Carlo passes, D and α values will be simulated independently, so that large D and α values (for site 1, for example) certainly can be simulated on any given pass. However, for the entire simulation, many small D values will be simulated with large α values, so that the inverse relationship will loosely hold for the site.

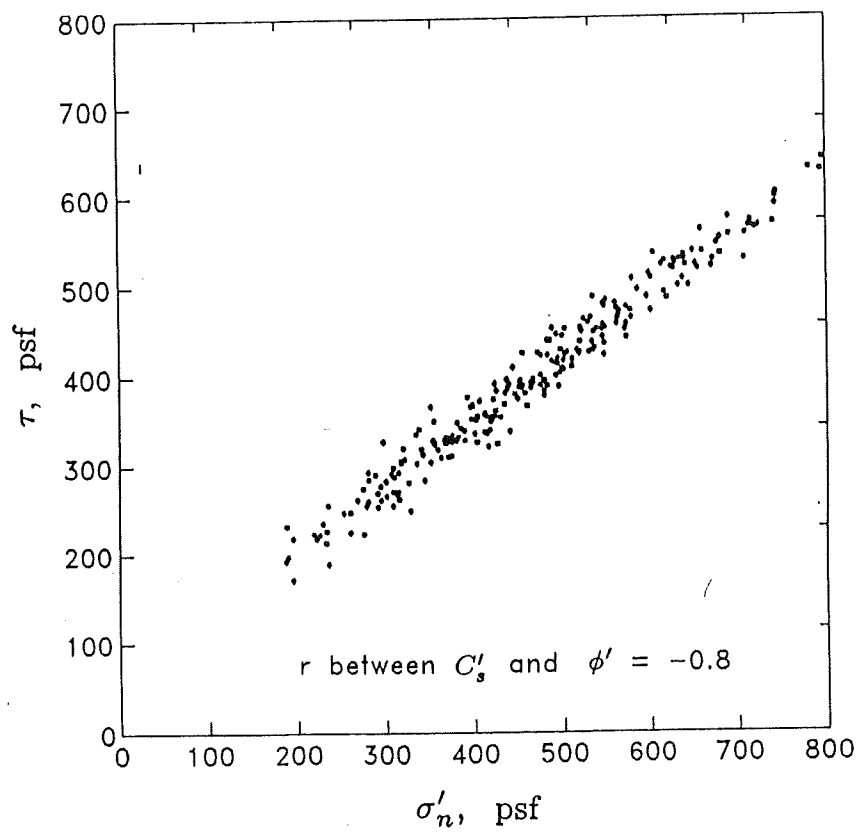
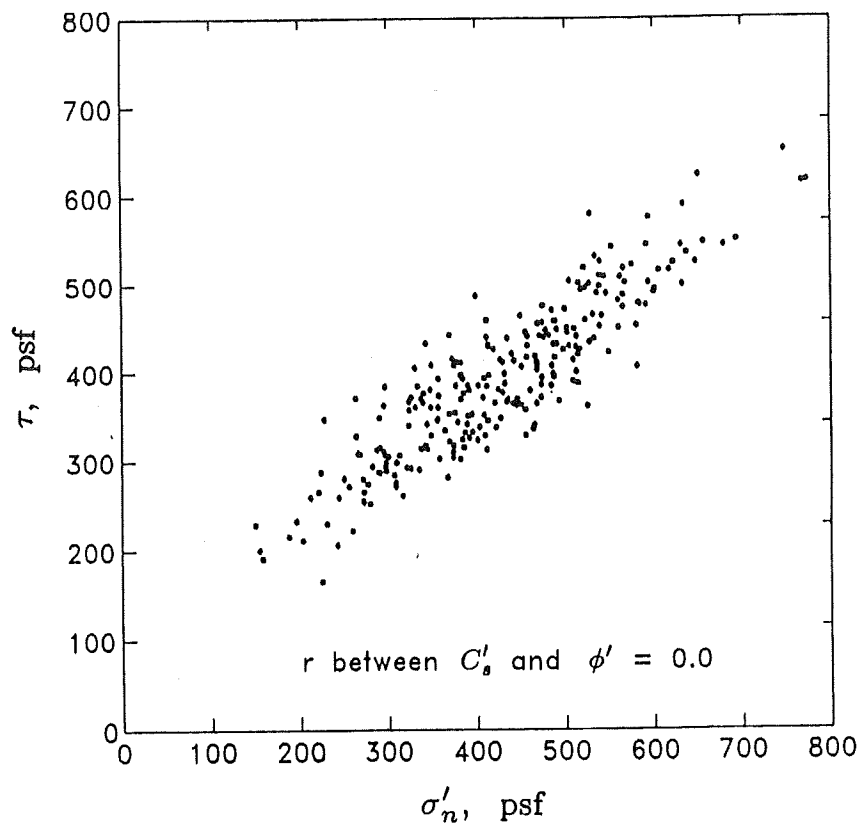


Figure 4.3—An inverse correlation between C'_s and ϕ' reduces the variance of soil shear strength.

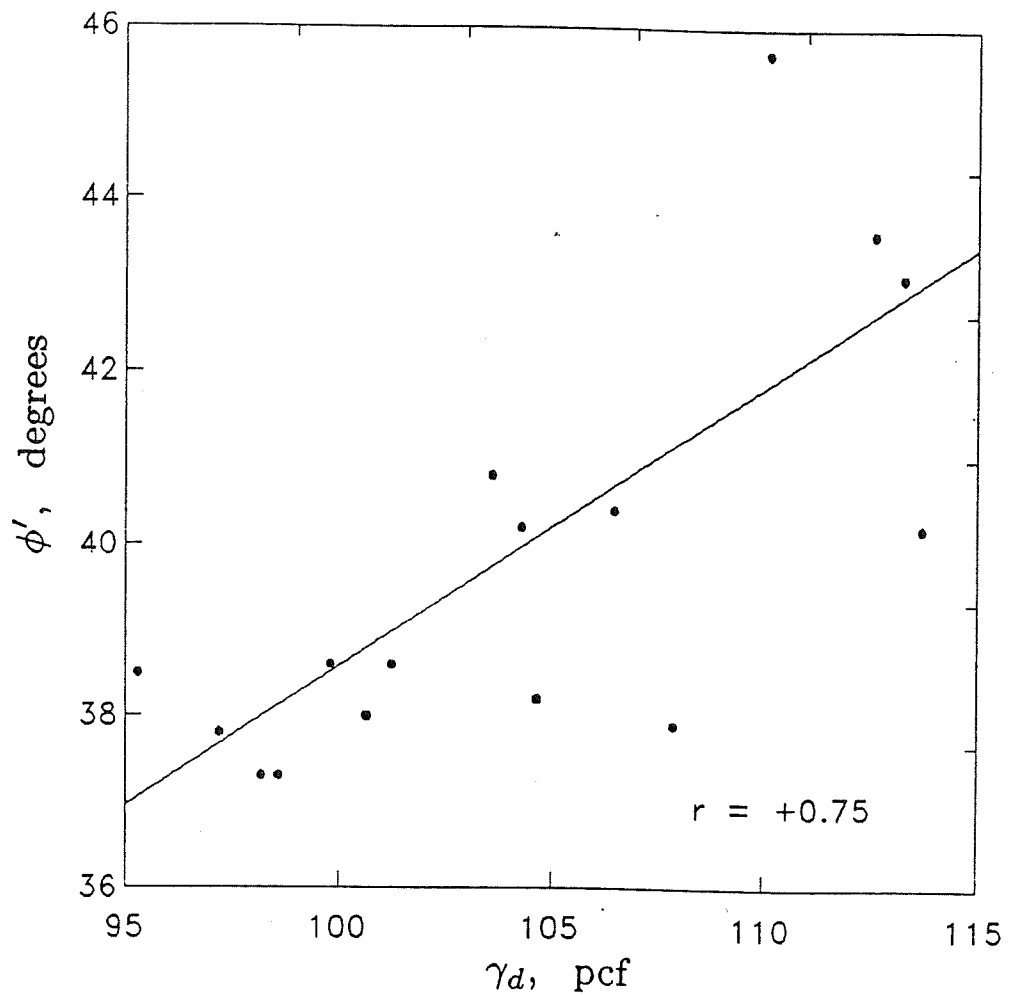


Figure 4.4—Example laboratory data illustrating a linear correlation between γ_d and ϕ' (data from Hammond and Hardcastle 1991).

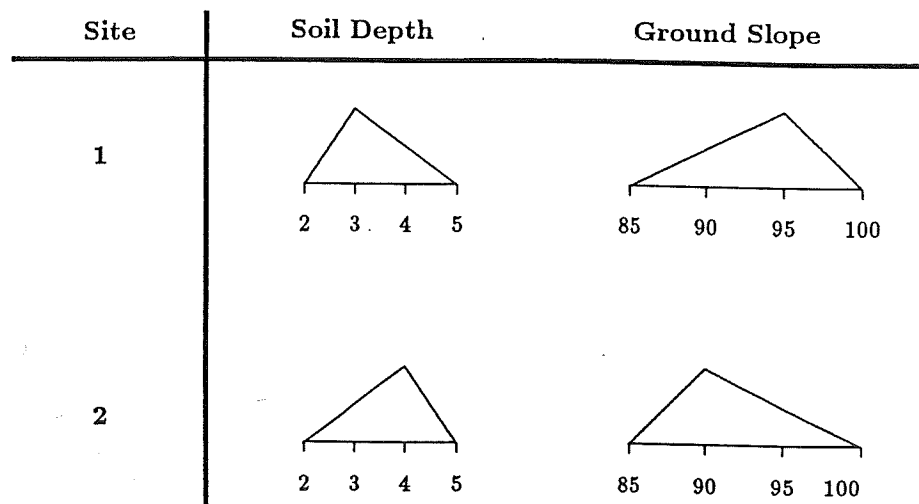


Figure 4.5—Distributions for two sites showing an inverse relationship between soil depth and ground slope.

Another method to account for a correlation between two variables on a given site is to analyze narrow enough classes for one variable so that within each class, the second variable can be considered to be independent of the first. The result is a conditional probability of failure for each class. For example, if slopes were analyzed in narrow classes, the results would be interpreted thus—"for areas of the site where the slope is between 45 and 55 percent, the probability of failure is 0.014, and for areas where the slope is between 55 and 65 percent, the probability of failure is 0.036." The specific locations of each class on the site would not have to be known to use this procedure.

The conditional probability of failure for each class of the first variable can be multiplied by the probability of the variable being in that class to give a weighted probability of failure. The weighted probabilities of failure for all classes then can be summed to give the average, or expected, probability of failure for the entire site. (Note that the probabilities of the variable being in each class must sum to 1.)

A future version of LISA may allow the user to enter a functional relationship between selected variables, thereby accounting for correlation in a more rigorous manner.

4.3 Simulating Groundwater Values

To prevent simulating a groundwater height (D_w) inconsistent with the simulated soil depth (D) on any given pass, LISA simulates a value of groundwater-soil depth ratio (D_w/D) from a distribution defined by the user. LISA then multiplies the simulated value of D_w/D by the simulated value of D to obtain a value of D_w to use in the infinite slope equation. Because the infinite slope model assumes a phreatic groundwater surface (see appendix A), LISA does not correctly calculate the FS if D_w/D values are negative or greater than 1, so effective stresses due to either capillary suction or artesian pressures cannot be analyzed. To prevent errors, LISA does not accept a distribution with D_w/D values that are negative or greater than 1.

4.4 Reproducibility of the Probability of Failure

If the user repeats a simulation with the same input PDF's but specifies a different seed number for the random number generator, LISA will simulate a different sequence of values for each random variable. This results in a different histogram of factors of safety and a slightly different value for the probability of failure. The more iterations (passes) used, the less the difference between simulation runs will be. The number of iterations required to provide consistent, stable results is a function of the shapes and ranges of the probability distributions used for each input variable.

Figure 4.6 illustrates how the variation between simulations decreases as the number of iterations in each simulation increases. In this example, 30 simulations were run with 100 iterations, then 30 simulations with 200 iterations, and so on up to 1,000 iterations, and the standard deviations of each set of the resulting 30 probabilities of failure computed. Clearly 100 iterations produce a large variation between simulation results, and the variation drops off rapidly with more than 200 iterations. *In order to produce stable results, we recommend that 1,000 iterations, the maximum allowed by LISA, be used for all production work. Even with 1,000 iterations, there will be some variation between simulations. Therefore, we also recommend that several simulation runs be performed using the same input distributions and different random seeds (those*

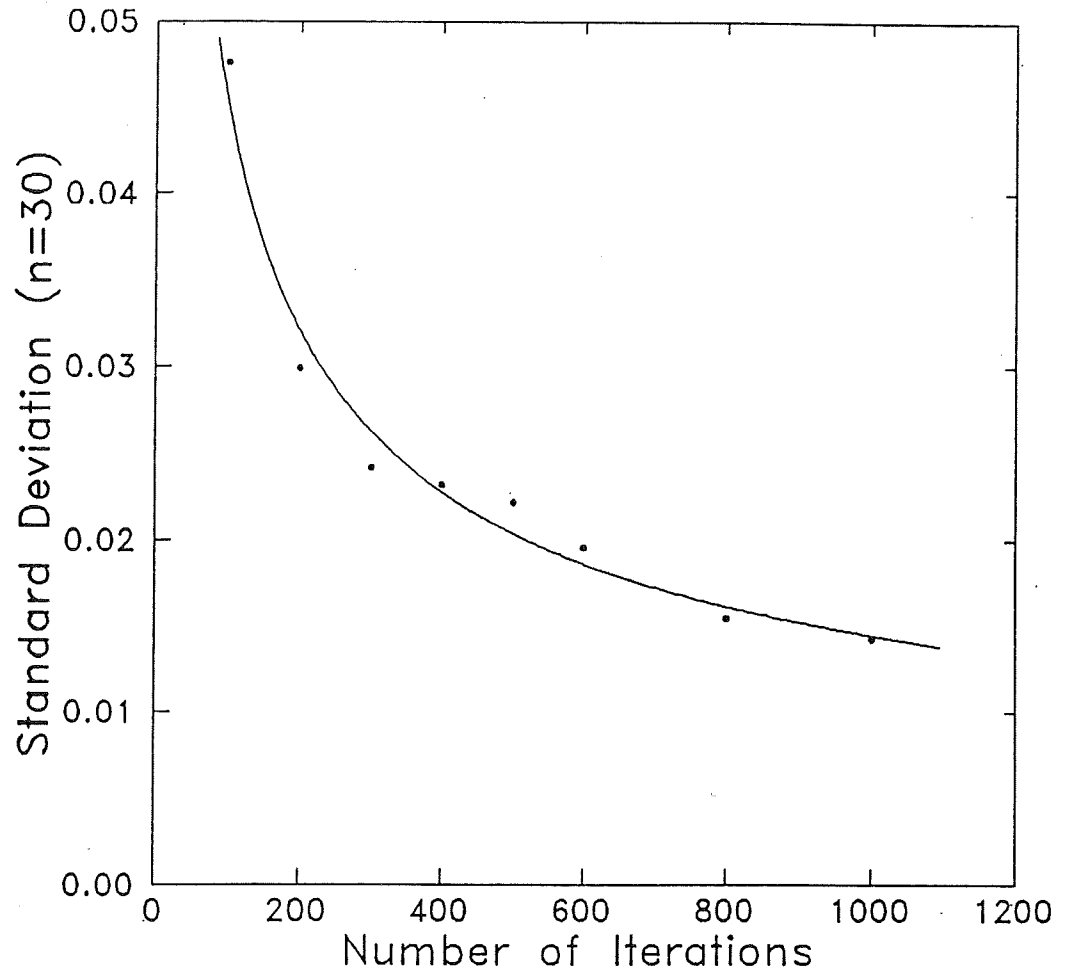


Figure 4.6—The standard deviations of 30 probability of failure values plotted against the number of iterations in each of the 30 simulations.

generated by LISA), and that the range of probability of failure values obtained be reported. This helps to reinforce the concept that LISA is a simulation that does not produce a unique “right” answer. Figure 4.7 illustrates typical amounts of variation in the probabilities of failure to expect from repeated simulations of 1,000 iterations. The amount of variation is proportionately larger for probabilities of failure that are smaller in magnitude, as demonstrated by the coefficients of variations.

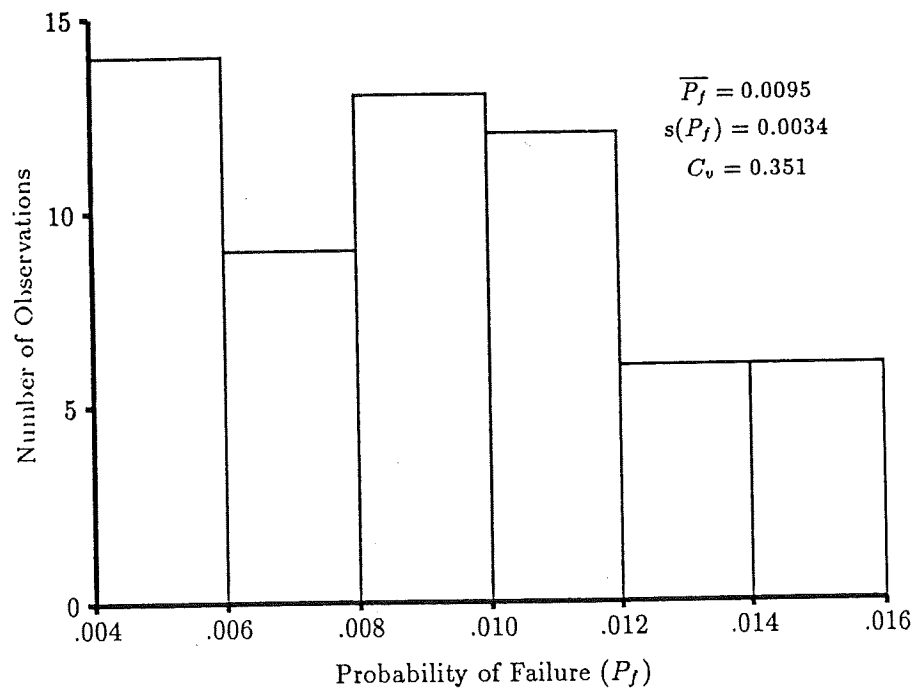
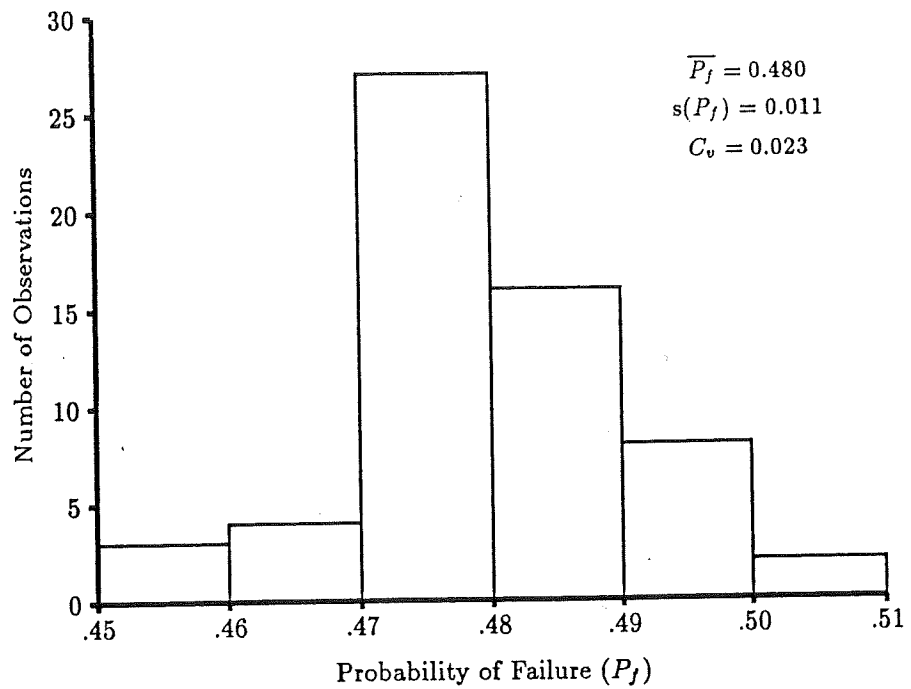


Figure 4.7—Illustrations of typical amounts of variation in probability of failure to expect with repeated simulations for two different sets of input distributions ($n = 60$).